Fredrik Rasten

nine *small pieces for* ***two*** *to* ***four*** *string players in* ***four****-part* ***five****-limit harmony*

(2020/21)

*'**nine** small pieces...'.* can be considered one piece or a collection consisting of 9 one-paged pieces, exploring 5-limit just intonation and *pitch drift* by the syntonic comma.

It is an extended version of the solo violin piece *Dyads* (2020). This extended version adds new harmony or counterpoint and it opens up for different combinations of string instruments.

Either one or both staffs can be played with double stops and one instrument per staff, or it can be realized with one instrument per voice (two per staff); or a combination.

Here are some options for different instrumentations:

String duo:

1st staff - violin (or viola)

2nd staff - cello

String trio 1:

1st staff - violin (or viola)

3rd voice (from top) - viola

4th voice (from top) - cello

String trio 2:

1st voice (from top) - violin (or viola)

2nd voice (from top) - violin (or viola)

2nd staff - cello

String quartet:

1st voice (from top) - violin

2nd voice (from top) - violin

3rd voice (from top) - viola

4th (voice from top) - cello

In the first staff, through combining groups of the *ptolemaic* (5-limit and pythagorean) dyads 5/3, 8/5, 5/4, 6/5, 3/2 and 4/3, notes are arrived at that produce more complex dyadic sounds when combined with the open (pythagorean) D or G-string.

These more complex "derivations" are accumulated through stacking of multiple ptolemaic intervals that "pulls" the comma in the same direction, and the resulting intervals are thus altered by two or three syntonic commas, from their pythagorean "origin".

Pythagorean tuning of **first staff** instruments (violin or viola):

This part is played only on the two lower strings of the violin or the two middle strings of the viola, tuned as a just pythagorean fifth (3/2):

D - 2 cents
G - 4 cents

Pythagorean tuning of **second staff** instruments (viola and / or cello):

A +/- 0 cents
D - 2 cents
G - 4 cents
C - 6 cents.

Notation:

- The accidentals belong to the Extended Helmholtz-Ellis JI Pitch Notation (developed by Marc Sabat and Wolfgang von Schweinitz)

Cent deviations from 12-tone equal temperament are written close to each note (except when already noted, and the same note is repeated).

Staff 1:

The staff notation for the first staff is supplemented with harmonic ratios above each dyad and melodic ratios between differing consecutive dyads. Note that the melodic ratios are written with respect to temporal movement:

4 : 5 means that the melodic movement *from* 4 *to* 5 constitutes both the harmonic *difference*, and the temporal *transition* between the two dyads.

- When the melodic ratios are graphically placed higher, this means that the melodic transition happens on the D-string, as when the melodic ratios are placed lower, this means that the melodic transition happens on the G-string.

Staff 2:

The second staff does not follow the logic as the first one, with simple dyads combining and stacking on each other to produce more complex ones. Moreover, it provides extra melodic and harmonic 'content' to most of the dyads produced in the first staff. Neither harmonic or melodic ratios are written into the score for this staff, however, the added overview of the ratios (in the end of this introduction part) will provide the information of the overall harmonic ratios for both staffs.

Interpretational notes:

The logic of the "bars" and the two staffs:

The first staff always starts alone while the second staff only comes in after a while, when the dyad in the first staff is well tuned and has sounded for some time. The first staff's dyad sounds out while the second staff's movement unfolds and eventually comes to the end of the bar. Then, the first staff's dyad is left sounding alone for some time before moving on to the next bar. This process is repeated, with the second staff coming in after a little while etc..

The only function of the bar lines is to make each of the dyads in staff one comprise its own *section* together with its corresponding counterpoint in staff two .

When two dyads are *not* connected with a notated bow, they should be separated by a few seconds of silence.

Each page should have some silence between them.

The piece can either progress in a normal succession or in any other order, and the performers are free to play a selection of pages instead of all of them.

The piece is played soft or medium soft throughout, but with a full tone and with clarity.

Con sordino if wanted.

Very small melodic movements, like 80:81 (the syntonic comma) and 125:128 (the lesser diesis), can *either* be performed with a subtle change like a glissando *or* as a more direct step.

Generally, slow glissandi between notes can occur occasionally, ad lib.

Optional metal chain:

As an option, **for instruments and realizations involving double stops**, a metal chain[3] - a necklace, key-chain or similar should be applied to the body of the instruments (or some of them) for some or all of the pages played (resting somewhere between the bridge and where the neck connects to the body), to create rattling texture.
To minimize the logistics of the chain being put in place and taken off, the intervals of using or not using the chain should generally be quite long.
The piece can also be realized with the chain applied throughout.

Ratios

Here is an overview of the harmonic ratios for each page.
The ratios are organized like this:

1 Like in the score, the first staff is on top, the second one below

2 . When a new number is introduced in a voice in **bold text,** this means that at this point there is an actual melodic step (or a transition from a pause to a note), whereas when a new number is introduced that is *not* in bold text, this means that the note continues unaltered, but now with a new numerical value in the overall harmonic ratio, due to a change in one of the other voices

3 . When a melodic change happens in just one of the voices and the change does not alter the numerical values of the other notes, only the change in that very voice is shown, whereas the other notes continue to sound as before, without new information regarding the harmony / ratio

4 . A line _ means a pause/silence for the given voice

5 . Some melodic / vertical 'commas', and other smaller melodic intervals internal to a voice are shown like this:

225 : 224, 75 : 72, 25 : 24

6 . On page 7 and 9, the ratios in parentheses are 7-limit ratios that the 5-limit harmony is very close to

It is only the voice with the melodic / vertical comma written in beside it that comprises this difference.

Besides showing this proximity *theoretically*, the performer of this voice can choose to
either play the comma after the 5-limit "approximation" has sounded for a while, and thereby enter the 7-limit ratio; letting this sound for a while; **or just remain** in the 5-limit without realizing the comma / prime limit-alteration

Page 1, bar 1-7:

20 **12**	12 6 9 36 **8** 4 6 24	**27** 20	8 24 8 32 16 **6** 18 6 24 12
15 **10**	9 **4** 6 **5** 20 6 3 **4** **15**	_ _	**5** 15 5 20 10 - **10** - **15** **9**

18 18 24 12 18 9 12 12 16 8 12 6	27 **16**	**15** 20 10 40 9 12 6 24
10 **8** **9** **5** **8** 4 9 9 12 6 9 **6**	18 12	**10** **15** **9** 36 _ **6** 8 4 **15**

Page 2, bar 1-7:

12 48 24 9 36 18	**30** 15 15 18 9 9	20 **15**	**16** 15
12 **45** **15** 5 20 10	15 **8** 8 10 5 **6**	_ **12** 8	12 8 **9**

20 15	60 45	30 **18**	**24** 12 4 18 9 3
12 9 **8** **9** **10**	**40** 30	20 **16** 15	16 8 3 15 **6** 2

Page 3, bar 1-8:

4 12 20 3 9 15	**15** 10	15 5 20 15 **12** 4 16 12	**10** **20** 10 20 9 18 9 18
3 **8** **12** 2 6 10	8 _ **6**	8 **3** 12 _ **8** 6 2 **9** _	6 **15** **8** **15** _ **4** 8 4 8

20 **15**	**45** 9 30 6	15 5 15 45 **12** 4 12 36	**10** 20 10 9 18 9
15 **16** **12** 8 **9** **10**	24 **5** **6** 20 4	10 **3** **8** 24 _ _ _ **16**	6 **15** **9** 4 8 4

Page 4, bar 1-7:

15 9 45 9 **10** 6 30 6	**5** 15 3 9	15 45 15 **12** **36** 12	**9** 18 8 16
_ 8 40 8 6 _ **18** **4**	_ **8** 2 6	8 24 8 _ **20** **6**	_ **6** 12 4 9 **15**

18 9 16 8	9 45 8 40	**25** 15 5 20 12 4
12 6 15 **5**	6 30 **32** 30 _ **24**	15 9 3 12 **8** **3** **2**

Page 5, bar 1-7:

9 **6**	18 72 **15** 60	**15** 30 12 24	12 **9**
6 4	12 **45** 8 **9** **10** 40	9 **10** 20 8 **15**	8 6 **5**

15 9	125 **64**	2 1
8 _ **6**	_ _	1 _ **1**

Page 6, bar 1-6:

12 **9**	**15** 5 9 3	15 45 30 **8** 24 16
8 8 **6**	8 **3** 6 2	9 **10** 30 20 6 20 **15**

24 16 24 16 **15** 10 15 10	**32** 25	5 20 5 20 **4** 16 4 16
16 **9** **16** **9** 12 8 12 8	_ _	3 12 3 12 _ **9** **2** **9**

Page 7, bar 1-8:

18 6 18 24 **15** 5 15 20	**5** 4	25 **16**	**16** 32 24 10 20 15
_ 8 **3** **10** **15**	_ 3	_ _	**9** 18 **16** _ **15** **12** 9

24 **20**	15 30 120 10 15 **12** 24 96 8 12	225 / (*14*) *225 : 224* 192 / (12)	18 **15**
16 **15** **16** 9	10 20 **75** *75 : 72* **6** 9 _ **5** **15** 60 5 **8**	144 / (9) 128 / (8)	_ **16** **10**

Page 8, bar 1-12:

10 **6**	10 20 **8** 16	32 64 32 24 48 24	32 16 20 10
8 **9** 5	9 **15** 5 10	_ **45** **20** **18** 15 30 15	18 9 15 **8**

20 10 10 5	20 **9**	5 15 5 20 **3** 9 3 12	**9** 18 9 8 16 8
9 **6** 8 4	12 8	3 **8** **3** 12 2 6 2 **9**	8 **9** **4** 6 12 6

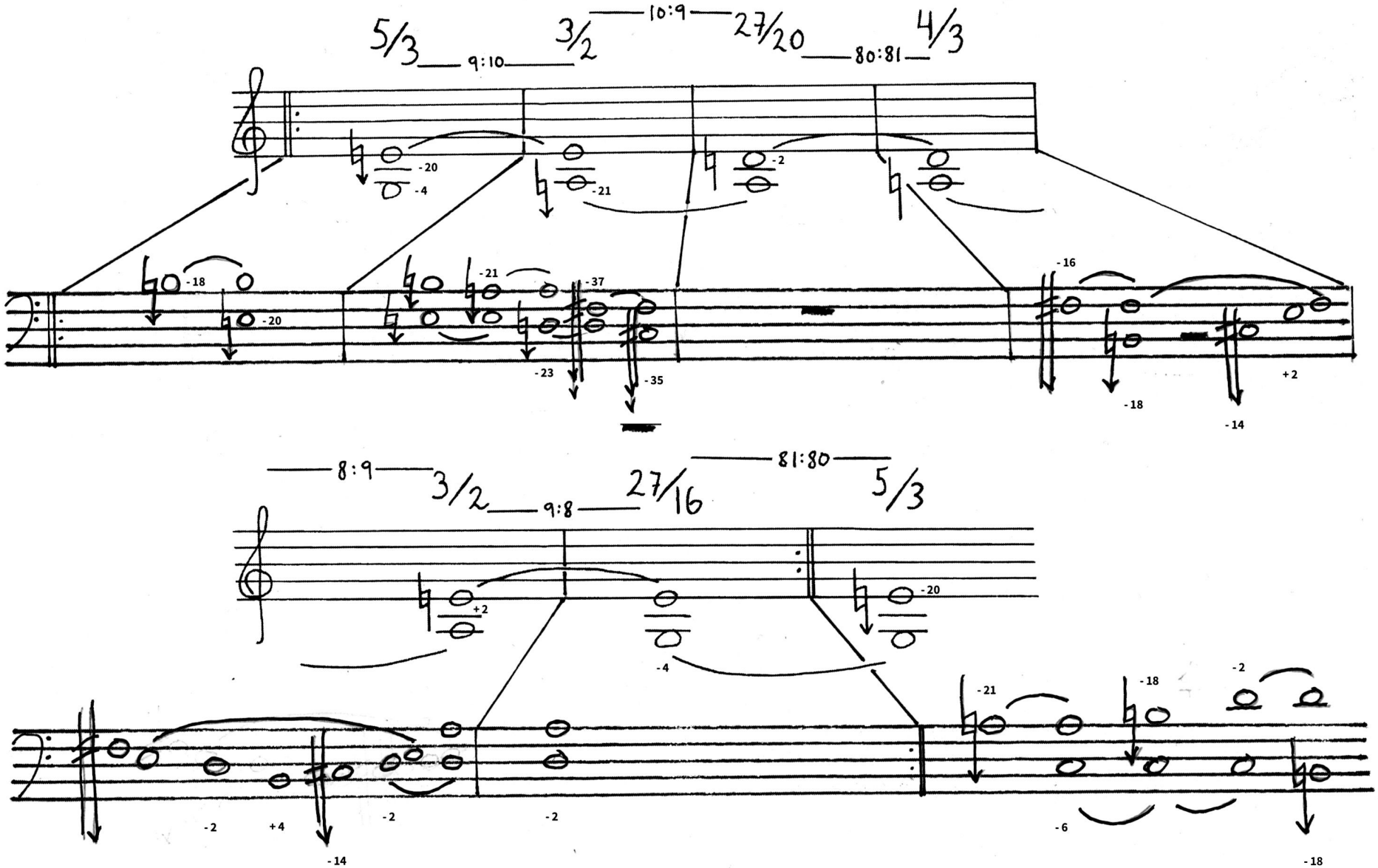
20 10 15 12 6 9	6 **5**	**81** 100	4 **5**
9 **4** 6 6 3 **4**	_ _	_ _	_ _

Page 9, bar 1-16:

16 32 8 24 **10** 20 5 15	12 24 15 120 15 **8** 16 10 80 10	**16** 24 10 15	384 / (12) **225 / (7)** *225 : 224*	384 / (12) 225 / (7) *225 : 224*
_ **12** 24 6 18 **9** **15** **3** **10**	9 **15** _ **75** *75:72* **9** 5 10 **6** 48 6	9 **16** 6 9 **18**	256 / (9) 288 / (8)	_ 256 / (9) 288 / (8)

15 **9**	384 / (12) **225 / (7)** *225 : 224*	15 **9**	384 / (12) **225 / (7)** *225 : 224*
10 **12**	256 / (9) **288 / (8)**	10 **8**	512 / (18) **288 / (8)**

15 **9**	**375** 256	6 3 4 2	**5** 15 20 3 9 12	384 / (12) **225 / (7)** *225 : 224*	**20** 12	**32** 8 32 16 20 5 20 10
20 **8** **9**	_ _	9 **4** _ _	6 18 24 **16** _ **8** **15**	_ **256 / (9)** **288 / (8)**	15 _ **9**	25 *25 : 24* **6** 24 12 **15** 9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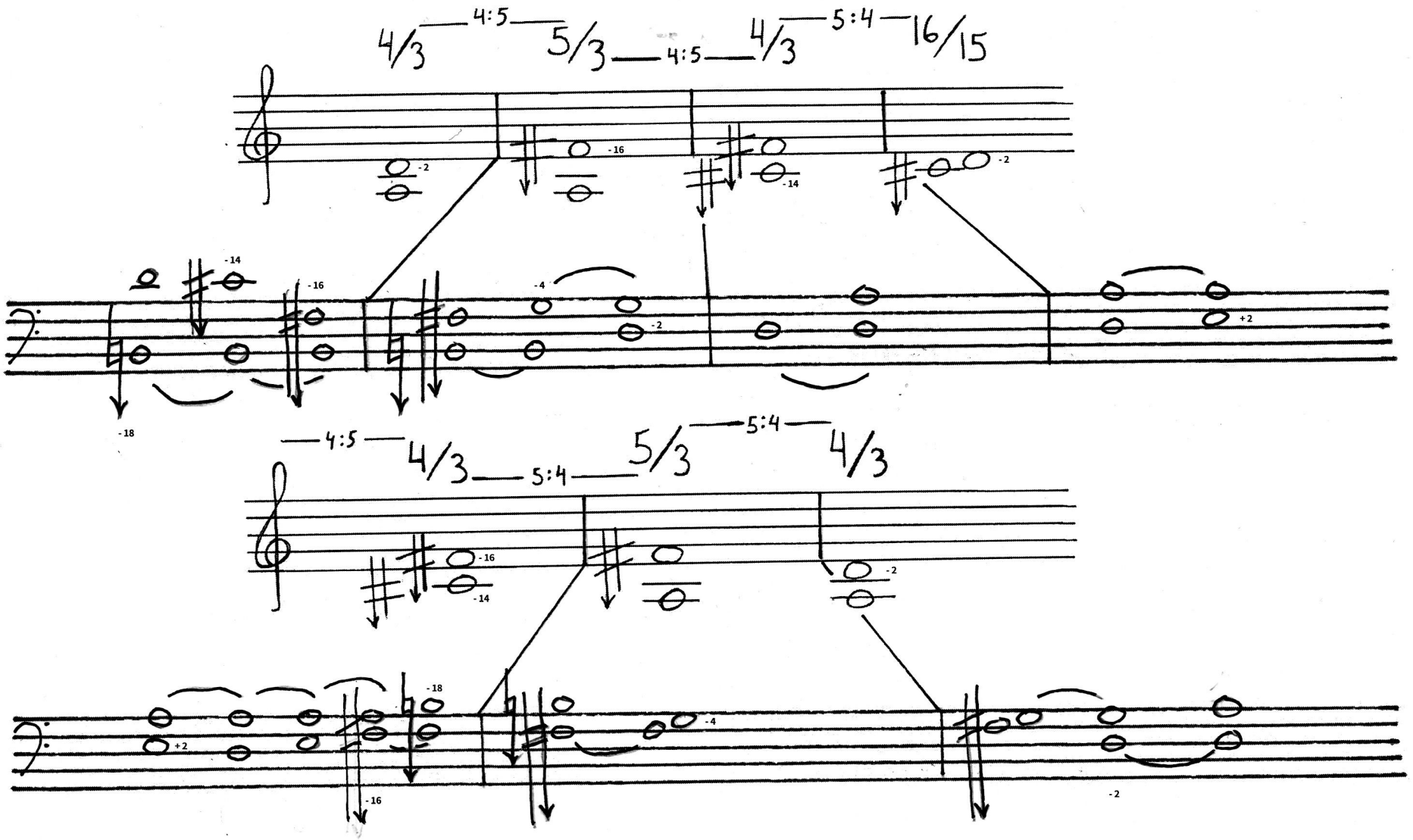
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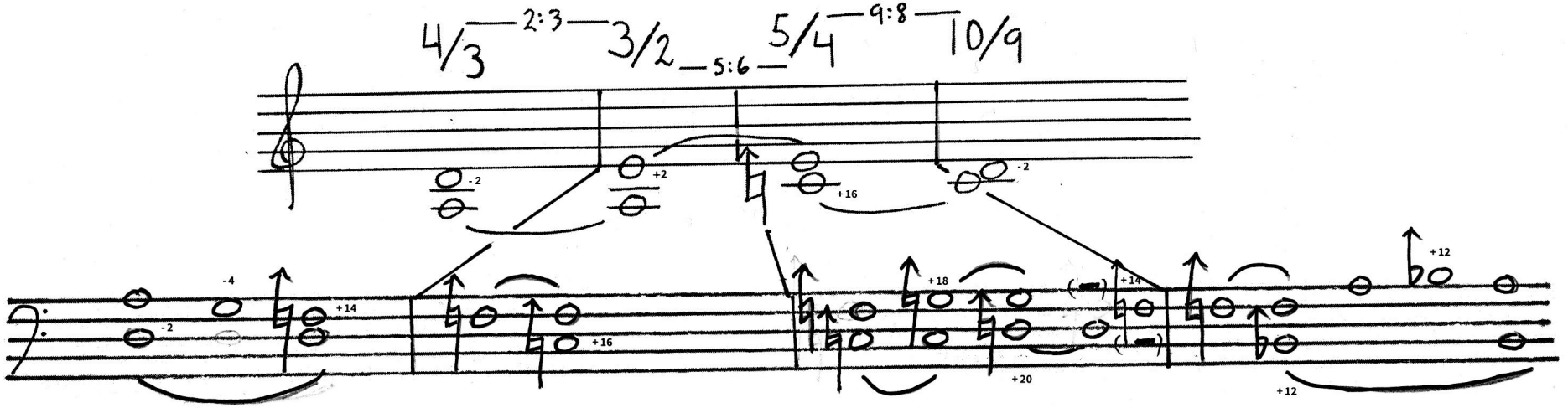
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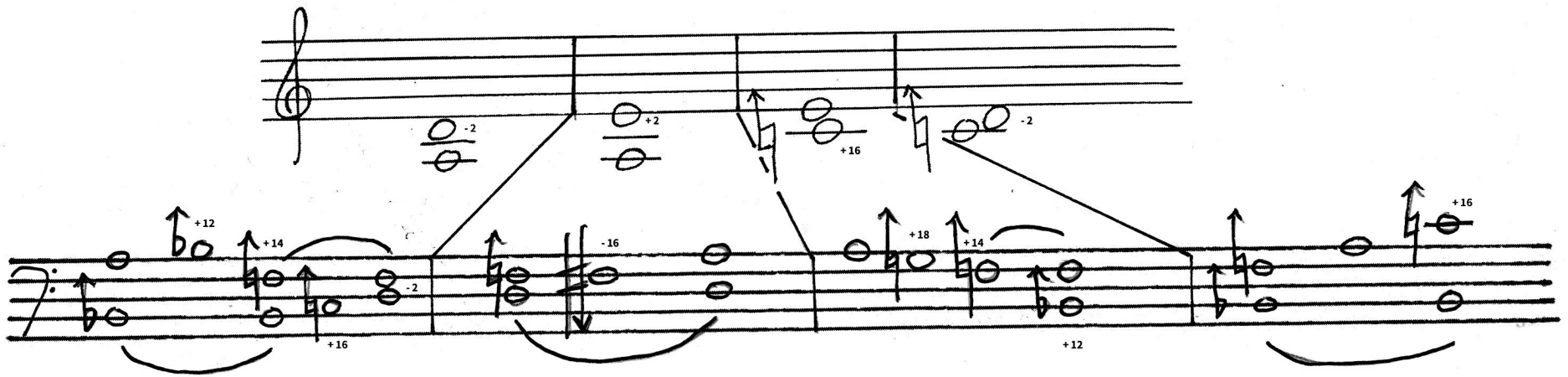

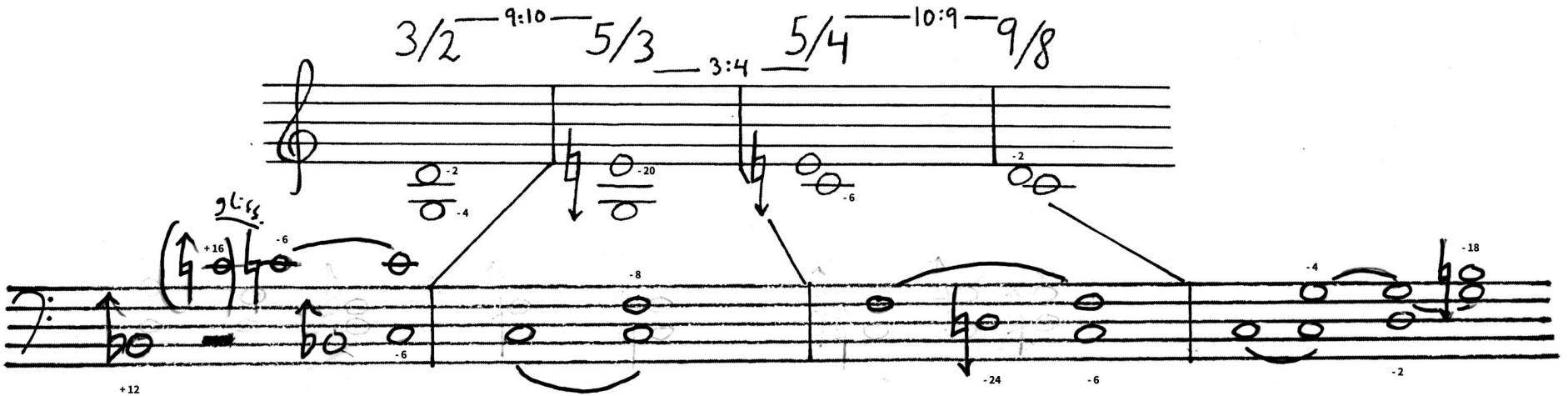
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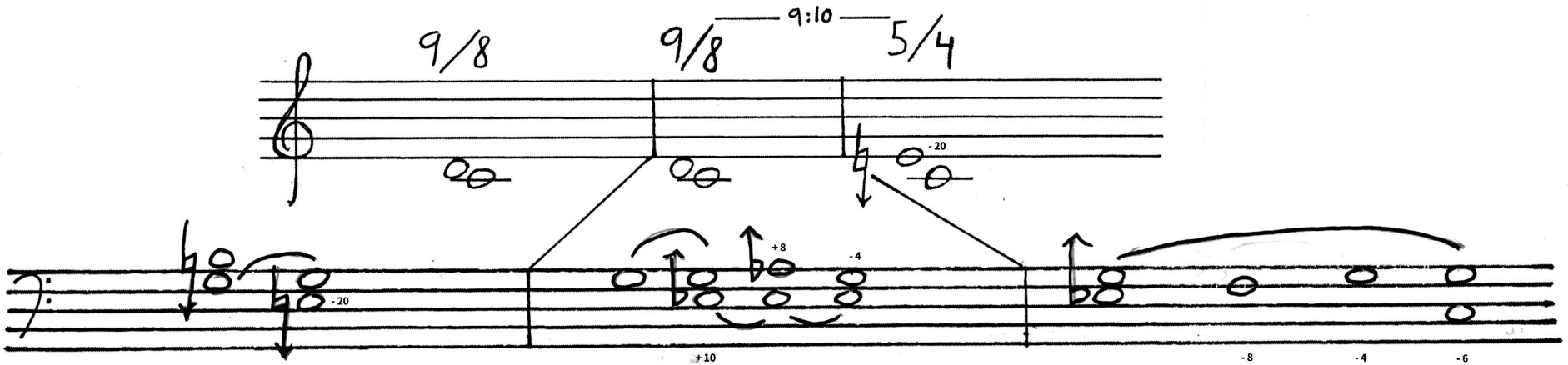
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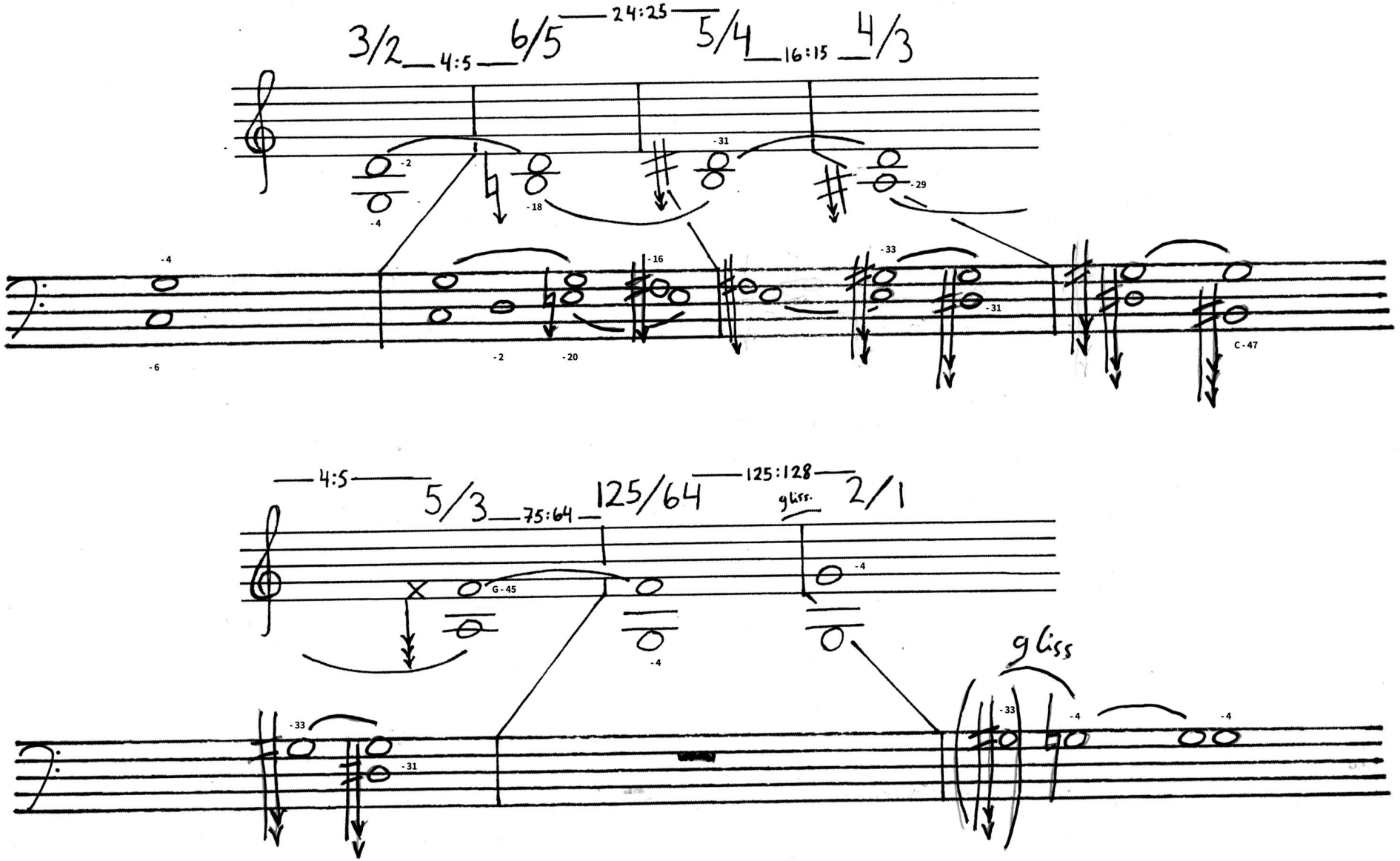
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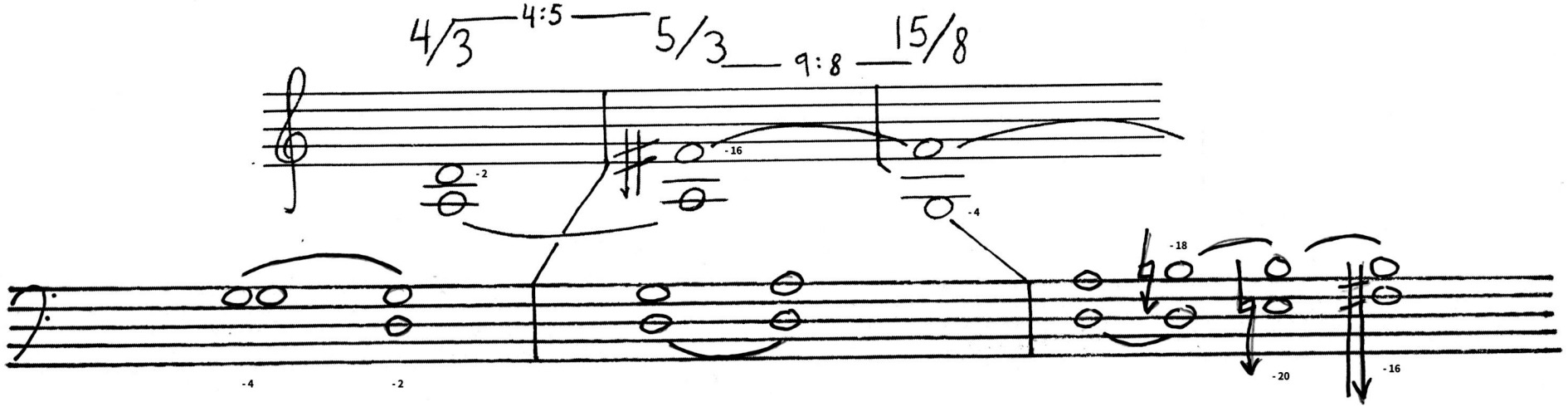
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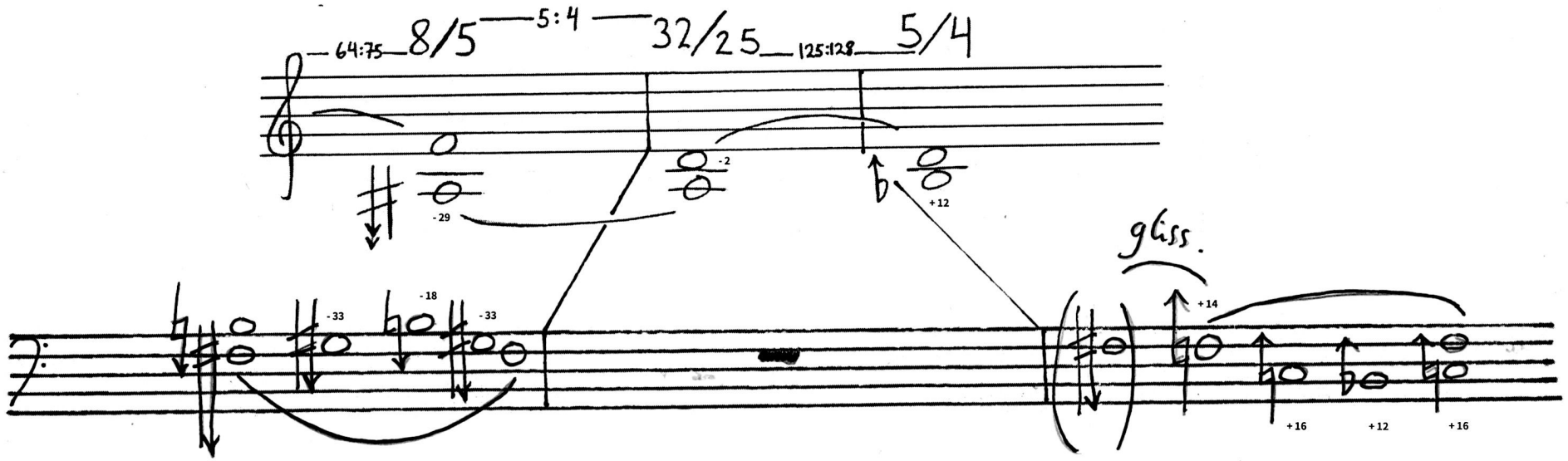
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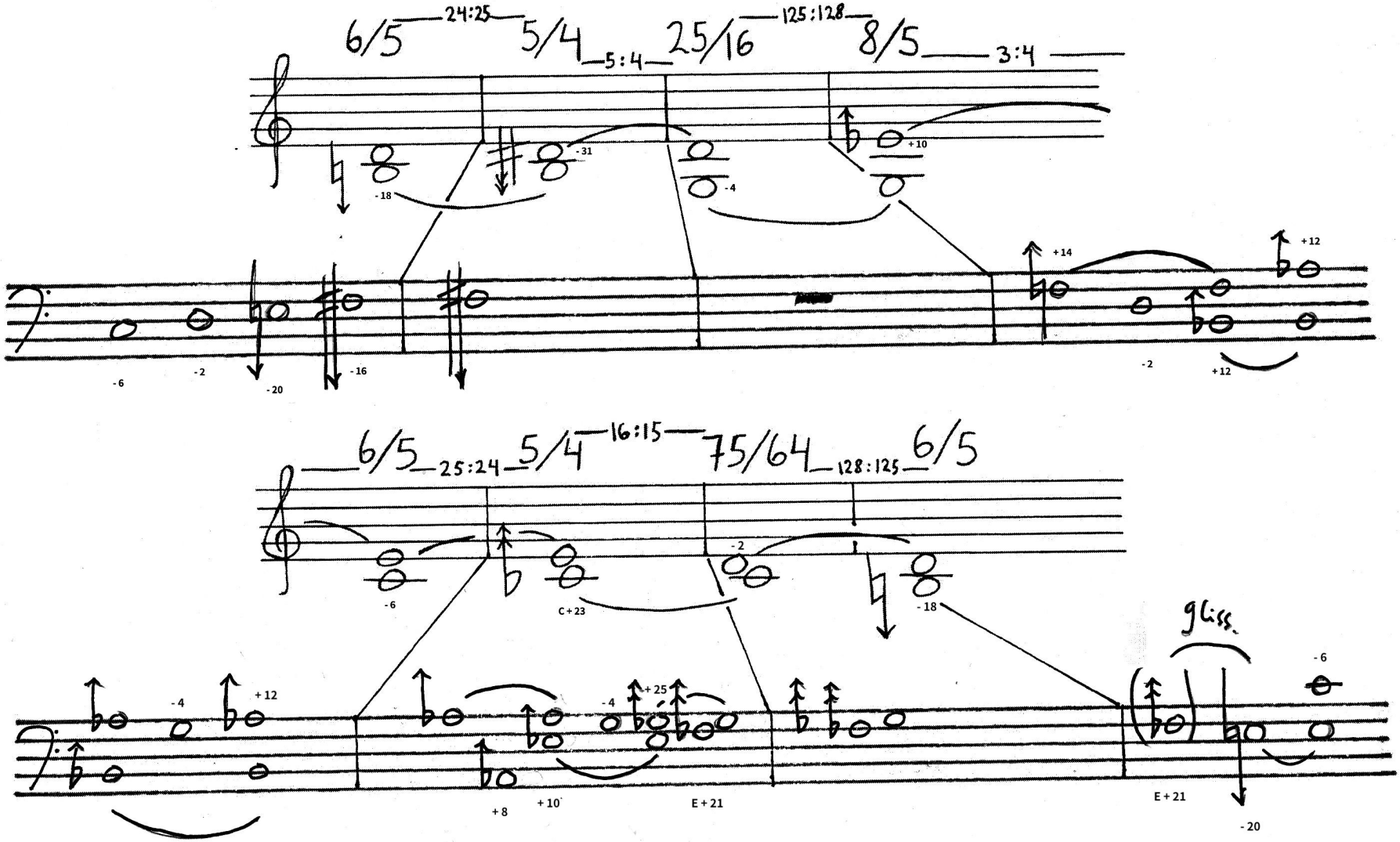
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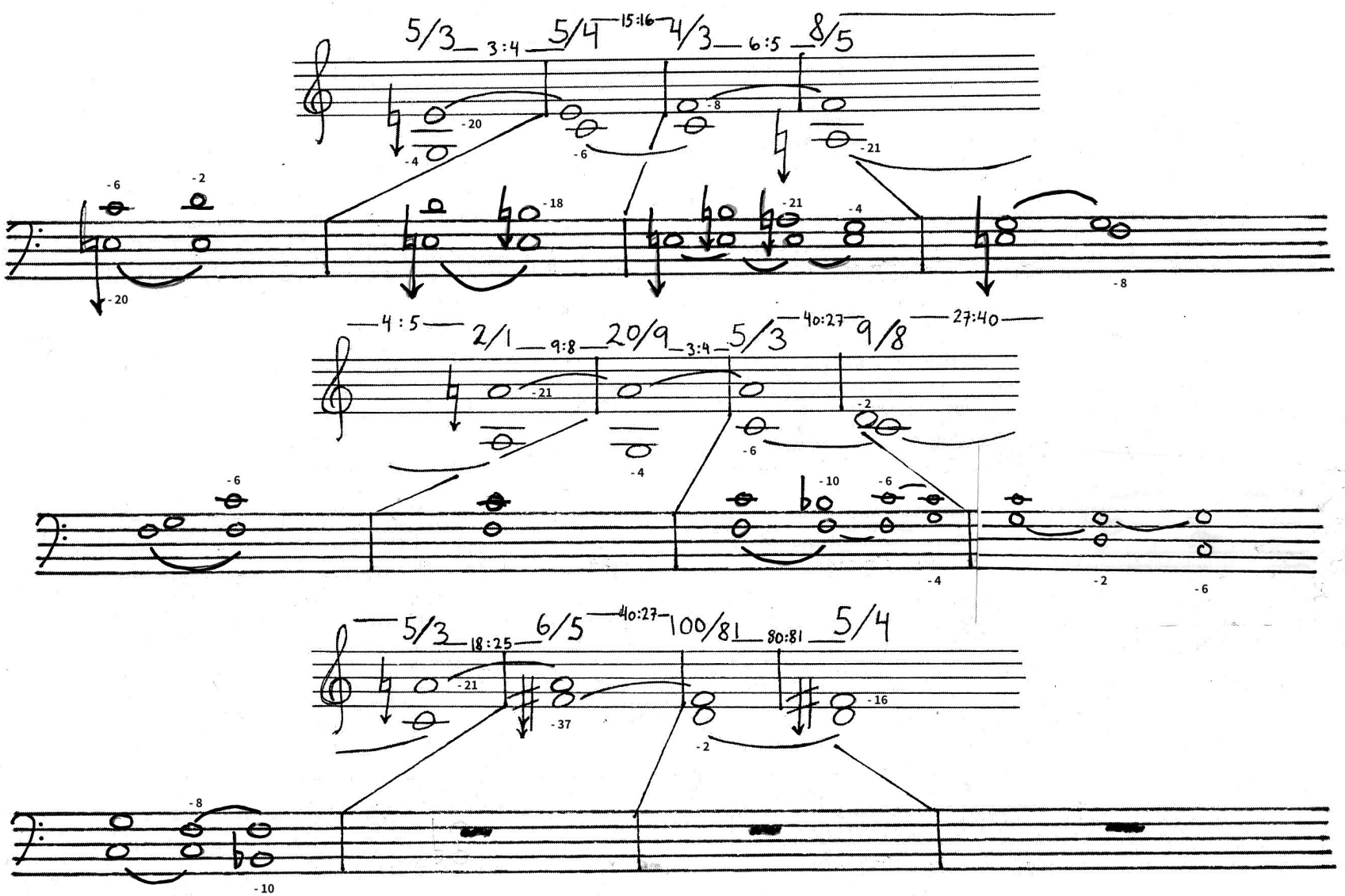
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G+37 -4 G+37 -4
C+35 B+23 F+33 A+20
A+20 B+23
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